

00 P1 A4	<b>4C, 1A, 5C</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> find derivatives and equate</li> <li>•<sup>2</sup> <math>3x^2 - 12</math> and <math>10x - 15</math></li> <li>•<sup>3</sup> <math>3x^2 - 10x + 3 = 0</math></li> <li>•<sup>4</sup> <math>x = 3, x = \frac{1}{3}</math></li> <li>•<sup>5</sup> tangents at <math>x = \frac{1}{3}</math> are parallel, at <math>x = 3</math> coincident or  <math>\left(\frac{1}{3}, -2\frac{26}{27}\right) \left(\frac{1}{3}, -12\frac{4}{9}\right)</math> leading to coinc. at B(3,-8) and  not coincident at other two pts</li> <li>•<sup>6</sup> <math>\int (\text{cubic} - \text{parabola})</math></li> <li>•<sup>7</sup> <math>\int_{-1}^3 \dots dx</math></li> <li>•<sup>8</sup> <math>\int (x^3 - 5x^2 + 3x + 9) dx</math> or equiv.</li> <li>•<sup>9</sup> <math>\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^3</math> or equiv.</li> <li>•<sup>10</sup> <math>21\frac{1}{3}</math></li> </ul>
00 P2 A4	<b>2C, 3B</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>ax(x - 4)</math></li> <li>•<sup>2</sup> <math>a = -1</math> <span style="margin-left: 100px;">ans: <math>y = 4x - x^2</math></span></li> <li>•<sup>3</sup> <math>\int_2^k (\text{function from } (a))</math></li> <li>•<sup>4</sup> <math>-\frac{1}{3}x^3 + 2x^2</math></li> <li>•<sup>5</sup> <math>-\frac{1}{3}k^3 + 2k^2 - \left(-\frac{8}{3} + 8\right)</math></li> </ul>
01 P2 Q6	<b>4C</b> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{x^4 - 4}{x^2}</math></li> <li>•<sup>2</sup> <math>x^2 - 4x^{-2}</math></li> <li>•<sup>3</sup> <math>\frac{1}{3}x^3 + c</math></li> <li>•<sup>4</sup> <math>\frac{-4x^{-1}}{-1}</math> <span style="margin-left: 100px;">ans: <math>\frac{1}{3}x^3 + 4x^{-1} + c</math></span></li> </ul>
01 P2 Q8	<b>7B</b>

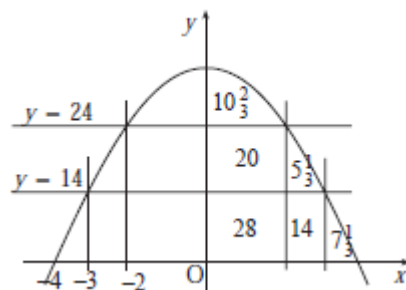
	<p>•<sup>1</sup> <math>y = x^3 - 3x^2 - x + 3</math></p> <p>•<sup>2</sup> <math>\int_1^4 [\dots] dx</math> or <math>\int_{-2}^1 [\dots] dx</math></p> <p>•<sup>3</sup> <math>\int [(5x - 5) - (x^3 - 3x^2 - x + 3)] dx</math> or <math>\int [(x^3 - 3x^2 - x + 3) - (5x - 5)] dx</math></p> <p>At this point candidates may tidy up and then integrate e.g.</p> <p>•<sup>4</sup> <math>\int (-x^3 + 3x^2 + 6x - 8) dx</math></p> <p>•<sup>5</sup> <math>\left[-\frac{1}{4}x^4 + x^3 + 3x^2 - 8x\right]</math></p> <p>or they may integrate and then tidy up after limits in e.g.</p> <p>•<sup>4</sup> <math>\left[\frac{5}{2}x^2 - 5x\right]</math></p> <p>•<sup>5</sup> <math>\left[-\frac{1}{4}x^4 - x^3 - \frac{1}{2}x^2 + 3x\right]</math></p> <p>•<sup>6</sup> <math>20\frac{1}{4}</math> or <math>-20\frac{1}{4}</math> depending on chosen integrals</p> <p>•<sup>7</sup> <math>40\frac{1}{2}</math></p>
02 P2 Q5	<p>6C</p> <p>•<sup>1</sup> <math>1 + 10x - 2x^2 = 1 + 5x - x^2</math></p> <p>•<sup>2</sup> <math>x = 0,5</math> and <math>\int_0^5 \left( \right)</math></p> <p>•<sup>3</sup> <math>\int ((1 + 10x - 2x^2) - (1 + 5x - x^2)) dx</math></p> <p>•<sup>4</sup> <math>\int (5x - x^2) dx</math></p> <p>•<sup>5</sup> <math>\frac{5}{2}x^2 - \frac{1}{3}x^3</math></p> <p>•<sup>6</sup> <math>20\frac{5}{6}</math></p>
4.(JA N) 02 P1	<p>4C</p> <p>•<sup>1</sup> <math>x^{\frac{4}{3}}</math></p> <p>•<sup>2</sup> <math>x^{-\frac{1}{2}}</math></p> <p>•<sup>3</sup> <math>\frac{3}{4}x^{\frac{4}{3}}</math> or <math>-2x^{\frac{1}{2}}</math></p> <p>•<sup>4</sup> <math>\frac{3}{4}x^{\frac{4}{3}} - 2x^{\frac{1}{2}} + c</math></p>
6.(JA N) 02 P2	<p>8A</p>

	<ul style="list-style-type: none"> <li>•<sup>1</sup> strategy : <math>f</math> to <math>x</math>-axis + <math>g</math> to <math>x</math>-axis + rectangle</li> <li>•<sup>2</sup> <math>\int_{-5}^0 \frac{1}{4}(-x^2 - 5x) dx</math></li> <li>•<sup>3</sup> <math>\left[-\frac{1}{12}x^3 - \frac{5}{8}x^2\right]_{-5}^0</math></li> <li>•<sup>4</sup> <math>5\frac{5}{24}</math> (5.2)</li> <li>•<sup>5</sup> <math>\int_0^5 \frac{1}{12}(x^2 - 5x) dx</math></li> <li>•<sup>6</sup> <math>\left[\frac{1}{36}x^3 - \frac{5}{24}x^2\right]_0^5</math></li> <li>•<sup>7</sup> <math>-1\frac{53}{72}</math> (-1.7) and <math>63\frac{17}{36}</math> (63.5)</li> <li>•<sup>8</sup> 6 kilowatts</li> </ul>
03 P2 Q3	<p><b>5C</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int \left( (x^2 + 2x) - (x^3 - x^2 - 6x) \right) dx</math> stated</li> <li>•<sup>2</sup> <math>\int_0^4 \dots</math></li> <li>•<sup>3</sup> <math>\int (8x + 2x^2 - x^3) dx</math></li> <li>•<sup>4</sup> <math>\left[ 4x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 \right]_0^4</math></li> <li>•<sup>5</sup> <math>42\frac{2}{3}</math></li> </ul>
04 P1 Q11	<p><b>3B, 5A</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>b = 2</math> or <math>y = ax(x - 2)</math></li> <li>•<sup>2</sup> substitute (1, -6)</li> <li>•<sup>3</sup> <math>a = 6</math></li> <li>•<sup>4</sup> <math>f(x) = \int (6x(x - 2)) dx</math></li> <li>•<sup>5</sup> <math>\int (6x^2 - 12x) dx</math></li> <li>•<sup>6</sup> <math>2x^3 - 6x^2</math></li> <li>•<sup>7</sup> <math>4 = 2 \times 1^3 - 6 \times 1^2 + c</math></li> <li>•<sup>8</sup> <math>c = 8</math></li> </ul> <p><b>OR</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> two simultaneous equations  <math>2a(2 - b) = 0</math> and <math>a(1 - b) = -6</math></li> <li>•<sup>2</sup> <math>b = 2</math></li> <li>•<sup>3</sup> <math>a = 6</math></li> </ul> <div style="display: flex; justify-content: space-between; align-items: flex-start;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y = k(x - 1)^2 - 6</math></li> <li>•<sup>2</sup> <math>0 = k(2 - 1)^2 - 6 \Rightarrow k = 6</math></li> <li>•<sup>3</sup> <math>y = 6(x - 1)^2 - 6 \Rightarrow y = 6x(x - 2)</math></li> </ul> <p><b>OR</b></p> </div>
04 P2 Q11	<b>8A</b>

	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x - \frac{1}{2}x^2 = 1 \cdot 5</math></li> <li>•<sup>2</sup> <math>x = 1, x = 3</math></li> <li>•<sup>3</sup> "split area up"      <i>stated or implied</i></li> <li>•<sup>4</sup> <math>\int \left(2x - \frac{1}{2}x^2 - \frac{3}{2}\right) dx</math></li> <li>•<sup>5</sup> <math>\int_1^3 \dots dx</math></li> <li>•<sup>6</sup> <math>\left[x^2 - \frac{1}{6}x^3 - \frac{3}{2}x\right]_1^3</math></li> <li>•<sup>7</sup> <math>\left(3^2 - \frac{1}{6} \cdot 3^3 - \frac{3}{2} \cdot 3\right) - \left(1^2 - \frac{1}{6} \cdot 1^3 - \frac{3}{2} \cdot 1\right)</math></li> <li>•<sup>8</sup> <math>\frac{2}{3}</math></li> </ul> <p><b>OR</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x - \frac{1}{2}x^2 = 1 \cdot 5</math></li> <li>•<sup>2</sup> <math>x = 1, x = 3</math></li> <li>•<sup>3</sup> <math>\int \left(2x - \frac{1}{2}x^2\right) dx</math></li> <li>•<sup>4</sup> choose limits, <math>a</math> and <math>b</math>: <math>0 \leq a \leq b \leq 4</math></li> <li>•<sup>5</sup> <math>\left[x^2 - \frac{1}{6}x^3\right]</math></li> <li>•<sup>6</sup> evaluate <math>\left[x^2 - \frac{1}{6}x^3\right]_a^b</math> for chosen values of <math>a</math> and <math>b</math></li> <li>•<sup>7</sup> state areas to be added/subtracted      <i>st / imp by</i> •<sup>8</sup></li> <li>•<sup>8</sup> <math>\frac{2}{3}</math></li> </ul>
05 P2 Q1	<p><b>4C</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>4x - x^{-2}</math></li> <li>•<sup>2</sup> <math>\frac{4x^2}{2}</math></li> <li>•<sup>3</sup> <math>-\frac{x^{-1}}{-1}</math></li> <li>•<sup>4</sup> <math>2x^2 + x^{-1} + c</math></li> </ul>
05 P2 Q5	<p><b>8C</b></p> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^2 = 2x^2 - 9</math></li> <li>•<sup>2</sup> <math>x = \pm 3</math></li> <li>•<sup>3</sup> <math>\int \text{upper} - \text{lower}</math></li> <li>•<sup>4</sup> <i>eg</i> <math>\int_0^3 \dots</math></li> <li>•<sup>5</sup> <math>x^2 - 2x^2 + 9</math></li> <li>•<sup>6</sup> <math>\left[-\frac{1}{3}x^3 + 9x\right]_0^3</math></li> <li>•<sup>7</sup> <math>\left(-\frac{1}{3} \times 3^3 + 9 \times 3\right) - 0</math></li> <li>•<sup>8</sup> <math>2 \times 18 = 36</math></li> </ul> </div> <div style="width: 45%;"> <ul style="list-style-type: none"> <li>•<sup>4</sup> <i>eg</i> <math>\int_{-3}^3 \dots</math></li> <li>•<sup>5</sup> <math>x^2 - 2x^2 + 9</math></li> <li>•<sup>6</sup> <math>\left[-\frac{1}{3}x^3 + 9x\right]_{-3}^3</math></li> <li>•<sup>7</sup> <math>\left(-\frac{1}{3} \times 3^3 + 9 \times 3\right) - \left(-\frac{1}{3} \times (-3)^3 + 9 \times (-3)\right)</math></li> <li>•<sup>8</sup> <math>36</math></li> </ul> </div> </div> <p style="text-align: center;"><b>OR</b></p>
06 P1 Q6	<b>4C, 3B</b>

	<p>•<sup>1</sup> <math>\int_0^1 (x^3 - 6x^2 + 4x + 1) dx</math> stated</p> <p>•<sup>2</sup> <math>\frac{1}{4}x^4 - \frac{6}{3}x^3 + \frac{4}{2}x^2 + x</math></p> <p>•<sup>3</sup> <math>\left(\frac{1}{4} \cdot 1^4 - 2 \cdot 1^3 + 2 \cdot 1^2 + 1\right) - 0</math></p> <p>•<sup>4</sup> <math>\frac{5}{4}</math> or equivalent</p> <p>•<sup>5</sup> <math>\int_1^2 \dots dx</math></p> <p>•<sup>6</sup> <math>\left(\frac{1}{4} \cdot 2^4 - 2 \cdot 2^3 + 2 \cdot 2^2 + 2\right) - \left(\frac{1}{4} \cdot 1^4 - 2 \cdot 1^3 + 2 \cdot 1^2 + 1\right) = -\frac{13}{4}</math></p> <p>•<sup>7</sup> <math>\frac{9}{2}</math> or equivalent <span style="float: right;">3</span></p>															
06 P2 Q5	<p><b>4C/B</b></p> <p>•<sup>1</sup> <math>y = \int \dots</math> stated or implied by •2</p> <p>•<sup>2</sup> <math>\frac{4}{2}x^2 - \frac{6}{3}x^3</math></p> <p>•<sup>3</sup> <math>9 = 2(-1)^2 - 2(-1)^3 + c</math></p> <p>•<sup>4</sup> <math>y = 2x^2 - 2x^3 + 5</math> stated explicitly</p> <p><b>OR</b></p> <p>•<sup>3</sup> <math>\begin{cases} y = 2x^2 - 2x^3 + c \\ \text{and} \\ 9 = 2(-1)^2 - 2(-1)^3 + c \end{cases}</math> stated explicitly</p> <p>•<sup>4</sup> <math>c = 5</math></p>															
07 P1 Q8	<p><b>1C, 3C, 5C/B</b></p> <p>•<sup>1</sup> <math>'f(3)' = 27 - 36 + 3 + 6 = 0</math></p> <p>•<sup>2</sup> <math>(x - 3)(x^2 \dots)</math></p> <p>•<sup>3</sup> <math>(x - 3)(x^2 - x - 2)</math></p> <p>•<sup>4</sup> <math>(x - 3)(x - 2)(x + 1)</math> so <math>A = (2, 0)</math></p> <div style="display: flex; align-items: center; justify-content: space-between;"> <div style="width: 45%;"> <p>•<sup>5</sup> <math>\int (x^3 - 4x^2 + x + 6) dx</math></p> <p>•<sup>6</sup> <math>\int_0^2</math></p> <p>•<sup>7</sup> <math>\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 + 6x</math></p> <p>•<sup>8</sup> <math>\frac{1}{4} \times 2^4 - \frac{4}{3} \times 2^3 + \frac{1}{2} \times 2^2 + 6 \times 2</math></p> <p>•<sup>9</sup> <math>\frac{22}{3}</math></p> </div> <div style="width: 45%;"> <p><i>Alt. Method 1 for •<sup>1</sup> to •<sup>4</sup></i></p> <table style="border-collapse: collapse; margin: 0 auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px 10px;">3</td> <td style="padding: 5px 10px;">1</td> <td style="padding: 5px 10px;">-4</td> <td style="padding: 5px 10px;">1</td> <td style="padding: 5px 10px;">6</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px 10px;"></td> <td style="padding: 5px 10px;"></td> <td style="padding: 5px 10px;">3</td> <td style="padding: 5px 10px;">-3</td> <td style="padding: 5px 10px;">-6</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px 10px;"></td> <td style="padding: 5px 10px;">1</td> <td style="padding: 5px 10px;">-1</td> <td style="padding: 5px 10px;">-2</td> <td style="padding: 5px 10px;"><u>0</u></td> </tr> </table> <p style="text-align: right; margin-top: 10px;">•<sup>1</sup></p> <p>•<sup>3</sup> <math>x^2 - x - 2</math></p> <p>•<sup>4</sup> <math>x = 2, x = -1</math> AND <math>x_A = 2</math></p> </div> </div>	3	1	-4	1	6			3	-3	-6		1	-1	-2	<u>0</u>
3	1	-4	1	6												
		3	-3	-6												
	1	-1	-2	<u>0</u>												
07 P2 Q10	<b>3C/B, 4A</b>															

	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>a = 2</math> and <math>b = 4</math> <b>or</b> <math>k(x-2)(x-4)</math></li> <li>•<sup>2</sup> <math>6 = k(0-2)(0-4)</math></li> <li>•<sup>3</sup> <math>k = \frac{3}{4}</math></li> <li>•<sup>4</sup> <math>\int \left( \frac{3}{4}(x-2)(x-4) \right) dx</math> s/i by •<sup>5</sup></li> <li>•<sup>5</sup> any two terms integrated correctly ( <math>\frac{3}{12}x^3</math> etc)</li> <li>•<sup>6</sup> <math>y = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + c</math></li> <li>•<sup>7</sup> <math>c = 6</math></li> </ul>
08 P2 Q7	<p>8A</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>32 - 2x^2 = 24</math> <b>or</b> 14</li> <li>•<sup>2</sup> <math>x = 2</math> and 3</li> <li>•<sup>3</sup> <math>\int (32 - 2x^2) dx</math></li> <li>•<sup>4</sup> <math>32x - \frac{2}{3}x^3</math></li> <li>•<sup>5</sup> <math>\left[ \dots \right]_2^3</math></li> <li>•<sup>6</sup> <math>19\frac{1}{3}</math></li> <li>•<sup>7</sup> e.g. <math>19\frac{1}{3} - 14 + 20</math> and then double s/i by •<sup>8</sup></li> <li>•<sup>8</sup> <math>50\frac{2}{3}</math></li> </ul> <p>2 For integrating "along the <math>y</math> - axis"</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> strategy: choose to integrate along <math>y</math>-axis</li> <li>•<sup>2</sup> <math>x = \sqrt{16 - \frac{1}{2}y}</math></li> <li>•<sup>3</sup> <math>\int \left( 16 - \frac{1}{2}y \right)^{\frac{1}{2}} dy</math></li> <li>•<sup>4</sup> <math>-2 \cdot \frac{2}{3} \left( 16 - \frac{1}{2}y \right)^{\frac{3}{2}}</math></li> <li>•<sup>5</sup> <math>\left[ \dots \right]_{14}^{24}</math></li> <li>•<sup>6</sup> <math>-\frac{4}{3} \left( 4^{\frac{3}{2}} - 0^{\frac{3}{2}} \right)</math></li> <li>•<sup>7</sup> <math>2 \times \dots</math></li> <li>•<sup>8</sup> <math>50\frac{2}{3}</math></li> </ul>



11 P2 Q4	<div> <div> <div>•<sup>1</sup> <math>\int \dots</math> or attempt integration</div> <div>•<sup>2</sup> Evidence of attempting to interpret the diagram to left of <math>y</math>-axis separately from diagram to the right.</div> <div> <div>•<sup>3</sup> <math>\int_{-2}^0</math></div> <div>•<sup>4</sup> <math>(x^3 - x^2 - 4x + 4) - (2x + 4)</math></div> <div>•<sup>5</sup> <math>\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2</math></div> <div>•<sup>6</sup> <math>-\left(\frac{1}{4}(-2)^4 - \frac{1}{3}(-2)^3 - 3(-2)^2\right)</math></div> <div>•<sup>7</sup> <math>\frac{16}{3}</math></div> <div>•<sup>8</sup> <math>\int_0^3 (2x + 4) - (x^3 - x^2 - 4x + 4) dx</math></div> <div>•<sup>9</sup> <math>\frac{63}{4}</math></div> <div>•<sup>10</sup> <math>21\frac{1}{12}</math> or <math>\frac{253}{12}</math> or <math>21 \cdot 1</math></div> </div> <div> <div>e.g. <math>\int_0^3</math> with no other</div> <div><math>(2x + 4) - (x^3 - x^2 - 4x + 4)</math></div> <div><math>3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4</math></div> <div><math>\left(3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4\right)</math></div> <div><math>\frac{63}{4}</math></div> <div><math>\int_{-2}^0 (x^3 - x^2 - 4x + 4) - (2x + 4) dx</math></div> <div><math>\frac{16}{3}</math></div> </div> </div> </div>	
12 P1 Q21	<div> <div> <div>know to use <math>x = 4</math></div> <div>complete evaluation</div> <div>state conclusion</div> <div>find quadratic factor</div> <div>factorise completely</div> <div>state solutions</div> </div> <div> <div>•<sup>1</sup> <math>4 \mid \begin{array}{rrrr} 1 &amp; -5 &amp; 2 &amp; 8 \\ &amp; &amp; &amp; \end{array}</math></div> <div>•<sup>2</sup> <math>4 \mid \begin{array}{rrrr} 1 &amp; -5 &amp; 2 &amp; 8 \\ &amp; 4 &amp; -4 &amp; -8 \\ \hline 1 &amp; -1 &amp; -2 &amp; 0 \end{array}</math></div> <div>•<sup>3</sup> 'remainder is zero so <math>(x - 4)</math> is a factor'</div> <div>•<sup>4</sup> <math>x^2 - x - 2</math> <span>stated, or implied by •<sup>5</sup></span></div> <div>•<sup>5</sup> <math>(x - 4)(x - 2)(x + 1)</math> <span>stated explicitly in any order</span></div> <div>•<sup>6</sup> <math>-1, 2, 4</math></div> <div>•<sup>1</sup> know to use <math>x = 4</math></div> <div>•<sup>2</sup> <math>64 - 80 + 8 + 8 = 0</math></div> <div>OR •<sup>3</sup> <math>(x - 4)</math> is a factor</div> </div> </div>	

	<p>identify <math>x_Q</math> from working in (a)</p> <p>interpret appropriate limits</p> <p>know and start to integrate</p> <p>complete integration</p> <p>substitute limits</p> <p>state area</p>	<ul style="list-style-type: none"> <li>•<sup>7</sup> 2</li> <li>•<sup>8</sup> 0, 2</li> <li>•<sup>9</sup> integrate one term correctly (but see Note 1</li> <li>•<sup>10</sup> <math>\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x</math> or equivalent</li> <li>•<sup>11</sup> <math>\left(\frac{1}{4}(2)^4 - \frac{5}{3}(2)^3 + 2^2 + 8 \times 2\right) - 0</math></li> <li>•<sup>12</sup> <math>\frac{32}{3}</math> or <math>10\frac{2}{3}</math> but not a decimal approximation</li> </ul>
13 P2 Q4	<p>know how to show that B is the point of intersection of the line and curve.</p> <p>know to integrate and interpret limits.</p> <p>use “upper – lower”</p> <p>integrate</p> <p>substitute limits</p> <p>evaluate area</p>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(-3)^3 + 3(-3)^2 + 2(-3) + 3 = -3</math> and <math>2(-3) + 3 = -3</math> or solving simultaneous equations</li> <li>•<sup>2</sup> <math>\int_{-3}^0 \dots \dots \dots</math></li> <li>•<sup>3</sup> <math>\int_{-3}^0 (x^3 + 3x^2 + 2x + 3) - (2x + 3) dx</math></li> <li>•<sup>4</sup> <math>\frac{1}{4} x^4 + x^3</math></li> <li>•<sup>5</sup> <math>0 - \left(\frac{1}{4}(-3)^4 + (-3)^3\right)</math></li> <li>•<sup>6</sup> <math>\frac{27}{4} \text{ units}^2</math></li> </ul>
14 P2 Q7	<p>know to and find intersection of line and curve</p> <p>use “upper – lower”</p> <p>integrate</p> <p>substitute limits and evaluate</p> <p>evaluate area developed</p> <p>set derivative to 2</p> <p>find point of contact</p> <p>find equation of road</p> <p>find correct integral</p> <p>calculate area</p>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x = 6x - x^2 \Rightarrow x = 0, x = 4</math></li> <li>•<sup>2</sup> <math>\int ((6x - x^2) - 2x) dx</math></li> <li>•<sup>3</sup> <math>2x^2 - \frac{1}{3}x^3</math></li> <li>•<sup>4</sup> <math>10\frac{2}{3}</math></li> <li>•<sup>5</sup> <math>10\frac{2}{3} \times 300 = 3200 \text{ m}^2</math></li> <li>•<sup>6</sup> <math>6 - 2x = 2</math></li> <li>•<sup>7</sup> <math>x = 2, y = 8</math></li> <li>•<sup>8</sup> <math>y = 2x + 4</math></li> <li>•<sup>9</sup> <math>\left[ (x^2 + 4x) - \left( 3x^2 - \frac{1}{3}x^3 \right) \right]_0^2</math></li> <li>•<sup>10</sup> <math>800 \text{ m}^2</math></li> </ul>



PRE 2000 ANSWERS – Indefinite Integrals

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{3}{4}x^4</math></li> <li>•<sup>2</sup> <math>2x^2</math></li> <li>•<sup>3</sup> <math>+c</math></li> </ul>
2	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{2}{3}x^3</math></li> <li>•<sup>2</sup> <math>+3x</math></li> <li>•<sup>3</sup> <math>+c</math></li> </ul>
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\left(\frac{x^2}{x\sqrt{x}}\right) x^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\left(\frac{-5}{x\sqrt{x}}\right) -5x^{-\frac{3}{2}}</math></li> <li>•<sup>3</sup> <math>\frac{x^{\frac{3}{2}}}{\frac{3}{2}}</math></li> <li>•<sup>4</sup> <math>\frac{-5}{-\frac{1}{2}}x^{-\frac{1}{2}}</math></li> </ul>

PRE 2000 ANSWERS – Definite Integrals

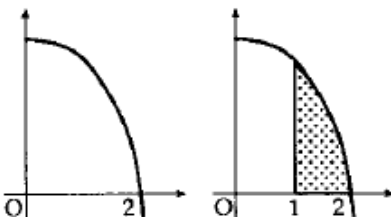
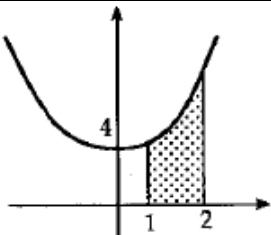
1	<ul style="list-style-type: none"> <li>•<sup>1</sup> know to expand brackets</li> <li>•<sup>2</sup> <math>x^4 + 2x + x^{-2}</math></li> <li>•<sup>3</sup> <math>\frac{1}{5}x^5 + x^2</math></li> <li>•<sup>4</sup> <math>-\frac{1}{x}</math></li> <li>•<sup>5</sup> <math>9\frac{7}{10}</math></li> </ul>
2	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>x^{\frac{3}{2}} + \frac{3}{2}</math></li> <li>•<sup>3</sup> <math>\frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)</math></li> <li>•<sup>4</sup> <math>\frac{14}{3}</math></li> </ul>
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>x^{-\frac{1}{2}}</math></li> <li>•<sup>3</sup> <math>\frac{2}{3}x^{\frac{3}{2}}</math></li> <li>•<sup>4</sup> <math>2x^{\frac{1}{2}}</math></li> <li>•<sup>5</sup> <math>21\frac{1}{3}</math></li> </ul>
4	<ul style="list-style-type: none"> <li>•<sup>1</sup> <i>strat:</i> know to divide</li> <li>•<sup>2</sup> <math>\frac{1}{2} + u^{-2}</math></li> <li>•<sup>3</sup> <math>\frac{1}{2}u</math></li> <li>•<sup>4</sup> <math>-u^{-1}</math></li> <li>•<sup>5</sup> <math>1</math></li> </ul>

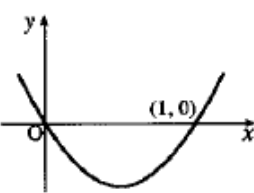
PRE 2000 ANSWERS – Differential Equations

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y = 2x^3 - x^2</math></li> <li>•<sup>2</sup> <math>y = 2x^3 - x^2 + k</math> and substituting</li> <li>•<sup>3</sup> <math>k = 5</math></li> </ul>
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2	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int (3x^2 + 1) dx</math></li> <li>•<sup>2</sup> <math>x^3 + x</math></li> <li>•<sup>3</sup> <math>+c</math></li> <li>•<sup>4</sup> <math>y = x^3 + x + 4</math></li> </ul>
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int (4x^3 - 1) dx = \dots\dots</math></li> <li>•<sup>2</sup> <math>x^4 - x</math></li> <li>•<sup>3</sup> <math>+c</math></li> <li>•<sup>4</sup> <math>f(2) = 14 + c</math></li> <li>•<sup>5</sup> <math>c = -15</math></li> </ul>
4	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\int (1 - 2x) dx = \dots\dots</math></li> <li>•<sup>2</sup> <math>x - x^2</math></li> <li>•<sup>3</sup> <math>+c</math></li> <li>•<sup>4</sup> <math>c = 3</math></li> </ul>
5	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^{-2}</math> <i>stated or implied by •<sup>2</sup> or •<sup>3</sup></i></li> <li>•<sup>2</sup> <math>y = \int (x^3 + x^{-2} - \frac{1}{4}) dx</math> <i>or</i> the appearance of any term of <math>\frac{1}{4}x^4 - \frac{1}{4}x - x^{-1}</math></li> <li>•<sup>3</sup> the remaining two terms</li> <li>•<sup>4</sup> <math>c = 3</math></li> </ul>

# PRE 2000 ANSWERS – Area under a Curve

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> evidence of e.g. triangle + rectangle</li> <li>•<sup>2</sup> area = 33</li> <li>•<sup>3</sup> <math>\int_2^5 (2x + 4) dx</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>4</sup> <math>x^2 + 4x</math></li> <li>•<sup>5</sup> <math>45 - 12 = 33</math></li> </ul>
2	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>4x</math></li> <li>•<sup>2</sup> <math>\frac{1}{3}x^3</math></li> <li>•<sup>3</sup> <math>1\frac{2}{3}</math></li> <li>•<sup>4</sup> for diagram 1 as shown</li> <li>•<sup>5</sup> shading 1 to 2</li> </ul>	
3	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^3</math></li> <li>•<sup>2</sup> <math>4x</math></li> <li>•<sup>3</sup> 11</li> <li>•<sup>4</sup> sketch of parabola with min above origin</li> <li>•<sup>5</sup> shade from 1 to 2</li> </ul>	

4	<p>(a) •<sup>1</sup> (1, 0)  •<sup>2</sup> (2, 0)</p> <p>(b) •<sup>3</sup> <math>\int f(x) dx</math>  •<sup>4</sup> <math>\int_0^1 -\int_1^2</math>  •<sup>5</sup> <math>(x+2)(x^2-3x+2)</math> or equiv.  •<sup>6</sup> <math>x^3 - x^2 - 4x + 4</math>  •<sup>7</sup> <math>\frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x</math>  •<sup>8</sup> <math>1\frac{11}{12}</math> or <math>-\frac{7}{12}</math>  •<sup>9</sup> <math>2\frac{1}{2}</math></p>
5	<p>(a) •<sup>1</sup> <math>18 - \frac{1}{8}x^2 = 0</math>  •<sup>2</sup> <math>x = \pm 12</math></p> <p>(b) •<sup>3</sup> <math>Area = 2 \int_0^{12} y \, dx</math>  •<sup>4</sup> integrating  •<sup>5</sup> 288  •<sup>6</sup> for knowing to subtract area of parabola from area of rectangle and multiply by 3.</p>
6	<div style="display: flex; justify-content: space-between;"> <div style="width: 60%;"> <p>(a) •<sup>1</sup> <math>f(x^2)</math> <i>stated or implied by</i> •<sup>2</sup>  •<sup>2</sup> <math>x^2 - 1</math>  •<sup>3</sup> <math>g(x-1)</math> <i>stated or implied by</i> •<sup>4</sup>  •<sup>4</sup> <math>(x-1)^2</math></p> </div> <div style="width: 35%;"> <p>(c) •<sup>8</sup> <math>\int_0^1 (2x^2 - 2x) \, dx</math>  •<sup>9</sup> <math>\left[\frac{2}{3}x^3 - x^2\right]</math>  •<sup>10</sup> <math>-\frac{1}{3}</math>  •<sup>11</sup> dealing with -ve</p> </div> </div> <p>(b) •<sup>5</sup> <math>(x-1)^2 + x^2 - 1</math> and complete proof  •<sup>6</sup> sketch as shown</p> <div style="text-align: center;">  </div> <p>•<sup>7</sup> minimum at <math>(\frac{1}{2}, -\frac{1}{2})</math> calculated or on sketch</p>

7

(a) •<sup>1</sup> (0,0)  
•<sup>2</sup> (3,0)

(b) •<sup>3</sup>  $f'(x) = 12x^2 - 24x$   
•<sup>4</sup>  $f'(x) = 0$  **stated explicitly**  
•<sup>5</sup>  $x = 0, x = 2$   
•<sup>6</sup>

$x$	$0^-$	$0$	$0^+$	$2^-$	$2$	$2^+$
$f'$	$+$	$0$	$-$	$-$	$0$	$+$

  
•<sup>7</sup> max at (0,0)  
•<sup>8</sup> min at (2,-16)

(c) •<sup>9</sup> correct shape  
•<sup>10</sup> (0,0),(3,0),(2,-16) annotated

(d) •<sup>11</sup>  $\int_0^3 (4x^3 - 12x^2) dx$   
•<sup>12</sup> area =  $-\int_0^3 (4x^3 - 12x^2) dx$   
•<sup>13</sup>  $[-x^4 + 4x^3]_0^3$   
•<sup>14</sup> 27

8

(a) •<sup>1</sup>  $m_{PQ} = -1$   
•<sup>2</sup>  $f(x) = 4x^{-2}$   
•<sup>3</sup>  $f'(x) = -8x^{-3}$   
•<sup>4</sup>  $-8x^{-3} = -1$   
•<sup>5</sup>  $x = 2$  and  $f(2) = 1$

(b) •<sup>6</sup>  $x + y = 3$   
•<sup>7</sup>  $\frac{4}{x^2} = 3$   
•<sup>8</sup>  $x \approx 1.15$   
•<sup>9</sup> A(1.15,3), B(3,0.44)

(c) •<sup>10</sup> suitable division of area  
•<sup>11</sup> rectangle  $OPA'C' = 3 \times 1.15 = 3.45$   
•<sup>12</sup> curved area  $QBA'C' = \int_{1.15}^3 \frac{4}{x^2} dx$   
•<sup>13</sup>  $\left[-\frac{4}{x}\right]_{1.15}^3$   
•<sup>14</sup> 2.15  
•<sup>15</sup>  $(3.45 + 2.15) \times 4 = 22.4$

9

(a) •<sup>1</sup>  $\frac{dy}{dx} = \dots\dots$   
•<sup>2</sup>  $\frac{1}{2}x^{-\frac{1}{2}}$   
•<sup>3</sup>  $m = \frac{dy}{dx}_{x=1} = \frac{1}{2}$   
•<sup>4</sup>  $y - 1 = \frac{1}{2}(x - 1)$   
•<sup>5</sup>  $P = (0, \frac{1}{2})$

(b) •<sup>6</sup> method for area of trapezium  
•<sup>7</sup>  $\frac{3}{4}$

(c) •<sup>8</sup>  $\int_0^1 x^{\frac{1}{2}} dx$   
•<sup>9</sup>  $\frac{2}{3}x^{\frac{3}{2}}$   
•<sup>10</sup>  $\frac{2}{3}$

(d) •<sup>11</sup> strategy: compare reduction with original  
•<sup>12</sup>  $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$  and  $\frac{\frac{1}{12}}{\frac{3}{4}} = \frac{1}{9}$   
•<sup>13</sup>  $\frac{1}{9} = 11.1\% > 10\%$  so objection correct

1	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x^2 = 4x - 2x^2</math> or <math>y = 4 - y</math></li> <li>•<sup>2</sup> <math>x = 1</math> and <math>x = -1</math></li> <li>•<sup>3</sup> <math>\int_{-1}^1 (4 - 2x^2 - 2x^2) dx</math></li> <li>•<sup>4</sup> <math>4x - \frac{2}{3}x^3 - \frac{2}{3}x^3</math></li> <li>•<sup>5</sup> <math>5\frac{1}{3}</math></li> </ul>
2	<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>A = (0, 5)</math></li> <li>•<sup>2</sup> <math>x_B = 2</math></li> <li>•<sup>3</sup> <math>y_B = 1</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>4</sup> <math>\int_0^4</math></li> <li>•<sup>5</sup> <math>\int \left( (5 + 4x - x^2) - (x^2 - 4x + 5) \right) dx</math></li> <li>•<sup>6</sup> <math>8x - 2x^2</math> or equiv.</li> <li>•<sup>7</sup> <math>4x^2 - \frac{2}{3}x^3</math> or equiv.</li> <li>•<sup>8</sup> <math>\frac{64}{3}</math></li> </ul> <p>(c)</p> <ul style="list-style-type: none"> <li>•<sup>9</sup> <math>n = -1</math></li> <li>•<sup>10</sup> <math>m = 10</math></li> </ul>
3	<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>p = 1</math></li> <li>•<sup>2</sup> <math>q = 2</math></li> </ul> <p style="text-align: center;">OR</p> <p>(b)</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>\int_0^1 ('OP' - 'OQ') dx + \int_1^2 ('PQ' - 'OQ') dx</math></li> <li>•<sup>4</sup> <math>\int_0^1 \left( x^2 + 3x - x + \frac{1}{4}x^2 \right) dx</math></li> <li>•<sup>5</sup> <math>\left[ \frac{5}{12}x^3 + x^2 \right]</math> or <math>\left[ \frac{1}{3}x^3 + \frac{3}{2}x^2 - \frac{1}{2}x^2 + \frac{1}{12}x^3 \right]</math></li> <li>•<sup>6</sup> <math>\frac{17}{12}</math></li> <li>•<sup>7</sup> <math>\int_1^2 \left( 4x^{-2} - x + \frac{1}{4}x^2 \right) dx</math></li> <li>•<sup>8</sup> <math>\left[ -4x^{-1} - \frac{1}{2}x^2 + \frac{1}{12}x^3 \right]</math></li> <li>•<sup>9</sup> <math>\frac{13}{12}</math> and Area = <math>2\frac{1}{2}</math></li> </ul> </div> <div style="width: 48%;"> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>\int_0^1 \dots dx + \int_1^2 \dots dx - \int_0^2 \dots dx</math></li> <li>•<sup>4</sup> <math>\int_0^1 (x^2 + 3x) dx + \int_1^2 (4x^{-2}) dx - \int_0^2 \left( x - \frac{1}{4}x^2 \right) dx</math></li> <li>•<sup>5</sup> <math>\left[ \frac{1}{3}x^3 + \frac{3}{2}x^2 \right]</math></li> <li>•<sup>6</sup> <math>[-4x^{-1}]</math></li> <li>•<sup>7</sup> <math>\left[ \frac{1}{2}x^2 - \frac{1}{12}x^3 \right]</math></li> <li>•<sup>8</sup> any two evaluations from <math>\frac{11}{6}, 2, \frac{4}{3}</math></li> <li>•<sup>9</sup> third evaluation and area = <math>\frac{11}{6} + 2 - \frac{4}{3} = 2\frac{1}{2}</math></li> </ul> </div> </div>

4	<div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> <p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> strat: <math>\frac{dy}{dx} = \dots</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 3x^2 - 2x - 6</math></li> <li>•<sup>3</sup> <math>m_{tgt} = -5</math></li> <li>•<sup>4</sup> <math>y + 8 = -5(x - 1)</math></li> <li>•<sup>5</sup> strat: attempt to simplify and equate <math>y</math>'s</li> <li>•<sup>6</sup> <math>x^3 - x^2 - x + 1 = 0</math></li> <li>•<sup>7</sup> strat: e.g. try to factorise</li> <li>•<sup>8</sup> <math>B = (-1, 2)</math></li> </ul> </div> <div style="width: 48%;"> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>9</sup> <math>\int (x^3 - x^2 - 6x - 2) - (-5x - 3) \, dx</math></li> <li>•<sup>10</sup> <math>\left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x \right]</math></li> <li>•<sup>11</sup> <math>1\frac{1}{3}</math></li> </ul> </div> </div>
5	<div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> <p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>a = 1</math></li> <li>•<sup>2</sup> <math>b = 6</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>\int_0^6 x(6 - x) \, dx</math></li> <li>•<sup>4</sup> <math>\int (6x - x^2) \, dx</math></li> <li>•<sup>5</sup> <math>3x^2 - \frac{1}{3}x^3</math></li> <li>•<sup>6</sup> 36</li> </ul> </div> <div style="width: 48%;"> <p>(c)</p> <ul style="list-style-type: none"> <li>•<sup>7</sup> <math>x = 6x - x^2</math></li> <li>•<sup>8</sup> <math>x_p = 5</math></li> <li>•<sup>9</sup> <math>\int_0^5 (6x - x^2 - x) \, dx</math> or equiv.</li> <li>•<sup>10</sup> <math>\left[ \frac{5}{2}x^2 - \frac{1}{3}x^3 \right]_0^5</math> or equiv.</li> <li>•<sup>11</sup> <math>\frac{125}{6}</math> or equiv.</li> </ul> </div> </div>
6	<div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> <p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> Area under curve - area under line</li> <li>•<sup>2</sup> abscissae at intersection are 0 and 12</li> <li>•<sup>3</sup> <math>\int_0^{12} \left( 4 + \frac{5}{3}x - \frac{1}{6}x^2 - \left( 4 - \frac{1}{3}x \right) \right) \, dx</math></li> <li>•<sup>4</sup> <math>\left[ x^2 - \frac{1}{18}x^3 \right]_0^{12}</math> or equivalent</li> <li>•<sup>5</sup> 48</li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>6</sup> <math>\int \dots \, dx = 24</math></li> <li>•<sup>7</sup> <math>\int_0^p \left( 2x - \frac{1}{6}x^2 \right) \, dx = 24</math> or equivalent statement</li> <li>•<sup>8</sup> <math>p^2 - \frac{1}{18}p^3 = 24</math></li> </ul> </div> <div style="width: 48%;"> <p>(c)</p> <ul style="list-style-type: none"> <li>•<sup>9</sup> "<math>f(6) = 0</math>" or equivalent</li> <li>•<sup>10</sup> divide by <math>(p - 6)</math></li> <li>•<sup>11</sup> <math>p^2 - 12p - 72</math></li> <li>•<sup>12</sup> <math>p = 6 \pm \sqrt{108}</math> or equivalent</li> <li>•<sup>13</sup> outside range 0 - 12</li> </ul> </div> </div>

7	<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> strategy: equate functions</li> <li>•<sup>2</sup> <math>x^4 = x</math></li> <li>•<sup>3</sup> <math>x(x^3 - 1) = 0 \Rightarrow x = 0, x = 1</math></li> <li>•<sup>4</sup> <math>\int x^{\frac{1}{2}} - x^2 dx</math></li> <li>•<sup>5</sup> <math>\int_0^k x^{\frac{1}{2}} - x^2 dx</math></li> <li>•<sup>6</sup> <math>\int_k^1 x^{\frac{1}{2}} - x^2 dx</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>7</sup> <math>\frac{2}{3}x^{\frac{3}{2}}</math></li> <li>•<sup>8</sup> <math>\frac{1}{3}x^3</math></li> <li>•<sup>9</sup> <math>\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3</math> or <math>\frac{2}{3} - \frac{1}{3} - \left(\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3\right)</math></li> <li>•<sup>10</sup> <math>\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3 = \frac{2}{3} - \frac{1}{3} - \left(\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3\right)</math> and completing proof</li> </ul> <p>(c)</p> <ul style="list-style-type: none"> <li>•<sup>11</sup> <math>2p^2 - 4p + 1 = 0</math></li> <li>•<sup>12</sup> strategy for solving: e.g. <math>p = \frac{4 \pm \sqrt{16 - 8}}{4}</math></li> <li>•<sup>13</sup> <math>p = 0.293, 1.707</math></li> <li>•<sup>14</sup> <math>k = 0.441</math></li> </ul>
8	<ul style="list-style-type: none"> <li>•<sup>1</sup> strategy: split into approp. parts</li> <li>•<sup>2</sup> <math>y = 1 \Rightarrow x = \pm 2</math></li> <li>•<sup>3</sup> first rectangular area</li> <li>•<sup>4</sup> <math>9 - \frac{1}{4}x^2</math> for integrand of shaped area</li> <li>•<sup>5</sup> <math>\int_2^5 dx</math> for limits of shaped area</li> <li>•<sup>6</sup> for integrating.....<math>\left(9x - \frac{1}{12}x^3\right)</math></li> <li>•<sup>7</sup> for evaluating.....<math>\left(\frac{56}{3}\right)</math></li> <li>•<sup>8</sup> total cross-sectional area = <math>\frac{208}{3}(m^2)</math></li> <li>•<sup>9</sup> volume = <math>4160 (m^3)</math></li> </ul>